Nuclear Magnetism and Electronic Order in ^{13}C Nanotubes

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Single wall carbon nanotubes grown entirely from ^{13}C form an ideal system to study the effect of electron interaction on nuclear magnetism in one dimension. If the electrons are in the metallic, Luttinger liquid regime, we show that even a very weak hyperfine coupling to the ^{13}C nuclear spins has a striking effect: The system is driven into an ordered phase, which combines electron and nuclear degrees of freedom, and which persists up into the millikelvin range. In this phase the conductance is reduced by a universal factor of 2, allowing for detection by standard transport experiments.

The physics of conduction electrons interacting with localized magnetic moments is central for numerous fields in condensed matter such as nuclear magnetism [1], heavy fermions [2], or ferromagnetic semiconductors [3, 4, 5, 6]. Nuclear spins embedded in metals offer an ideal platform to study the interplay between strong electron correlations and magnetism of localized moments in the RKKY regime. In two dimensions the magnetic properties of the localized moments [7, 8] depend indeed crucially on electron-electron interactions [9, 10, 11, 12, 13]. In one-dimensional (1D) systems such as single wall carbon nanotubes (SWNTs) electron correlations are even more important. For metallic (armchair) SWNT they lead to Luttinger liquid physics [14, 15, 16]. Recently, SWNTs made of ^{13}C, forming a nuclear spin lattice, have become experimentally available [17, 18, 19, 20]. Motivated by this we study here nuclear magnetism in metallic ^{13}C SWNTs. We show that even a weak hyperfine interaction can lead to a helical magnetic order of the nuclear spins (see Fig. 1) coexisting with an electron density order that combines charge and spin degrees of freedom. The ordered phases stabilize each other, and the critical temperature undergoes a dramatic renormalization up into the millikelvin range due to electron-electron interactions. In this new phase the electron spin susceptibility becomes anisotropic and the conductance of the SWNT drops by a universal factor of 2.

The drastic restructuring of the electron wave functions through the renormalization is very different from the case of two [7, 8] or three dimensions [1] where it is, in comparison, weak. The same renormalization leads to considerable anisotropy in the electron system: The nuclear magnetic field spontaneously breaks the spin rotational symmetry; it rotates in a plane, which we can associate with the spin (x, y) directions (see Fig. 1). This plane is singled out as an easy-plane through the stabilization of the electron density wave, and electron correlation functions become anisotropic between the spin (x, y) plane and the spin z direction. We illustrate this behavior below through the calculation of the electron spin susceptibilities. We emphasize that this anisotropy is a crucial feature of the SWNT system studied here and appears *spontaneously* due to strong renormalization of the RKKY interactions. This distinguishes our system, in particular, from models with built-in easy-axis anisotropy [21].

Model. — We assume that the electrons are confined in a single mode $\psi_\perp$ in the directions perpendicular to the tube axis. The nuclear spins $I = 1/2$ of the ^{13}C ions on a circular cross section have identical overlaps with this transverse mode, and so identical couplings to the electrons. Through their indirect RKKY interaction over the electron gas they are therefore locked in a ferromagnetic alignment (see Fig. 1). This RKKY interaction, described below, overrules furthermore the direct dipolar interaction between the nuclear spins. The latter is very small [22], $\sim 10^{-11}$ eV, and shall be neglected henceforth. This allows us to treat the nuclear spins as a 1D chain of *large* $\tilde{I} = IN_\perp$ spins, composed of the sum of the $N_\perp \sim 50$ spins around a circular cross section. Due to this, Kondo physics, which requires small quantum spins, can be excluded from the beginning.

Hence, we model the SWNT by a 1D nuclear spin lattice of length L coupled through the hyperfine interaction to a 1D electron gas. The Hamiltonian resembles that of a Kondo lattice $H = H_{el} + A\sum_i \hat{\mathbf{S}}_i \cdot \hat{\mathbf{I}}_i$, where i runs over the 1D lattice sites with positions r_i, $\hat{\mathbf{I}}_i = (\hat{I}_i^x, \hat{I}_i^y, \hat{I}_i^z)$ is the effective nuclear spin of size $\tilde{I} = IN_\perp$, $\hat{\mathbf{S}}_i = (\hat{S}_i^x, \hat{S}_i^y, \hat{S}_i^z)$ is the electron spin operator at site i, and $A = A_0/N_\perp$ is the on-site hyperfine interaction constant A_0 weighted by the transverse electron mode. In contrast to the usual Kondo lattice model, H_{el} describes the *interacting* electrons and is defined in Eq. (2) below.

The precise value of A_0 in SWNTs is unknown. Estimates in the literature [23] provide values of $A_0 \sim 10^{-7} - 10^{-6}$ eV, depending much on the curvature of the nanotube (higher values have been reported in [17] though). This compares with the typical energy scales of the electrons, which can be quantified by the value $E_F = v_F k_F/2$ (we set $\hbar = 1$ throughout this paper),

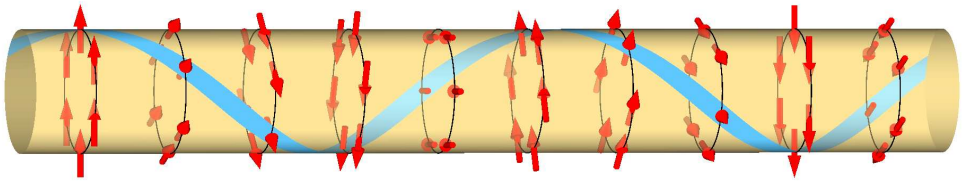

FIG. 1: Illustration of the helical nuclear magnetism (indicated by the blue ribbon) of the single wall ^{13}C nanotube (SWNT), which emerges below a critical temperature through strong renormalization of the hyperfine coupling by electron correlations. The nuclear spins (red arrows) order ferromagnetically on a cross-section of the SWNT and rotate along the SWNT axis with a period π/k_F in the spin xy-plane (chosen here arbitrarily orthogonal to the SWNT axis). Through the feedback of the nuclear magnetization the electric conductance of the SWNT is reduced by a factor of precisely 2.

where $k_F/\pi = n_{el}$ is the electron density in the system and v_F ($\approx 8\times 10^5$ m/s in SWNTs [14, 15, 24]) is the typical velocity of electron excitations. Through the dependence on n_{el}, E_F can vary between the meV to eV range.

Effective model. — Due to the small ratio A/E_F, the energy and time scales related to the electrons and nuclear spins decouple, and we can treat both subsystems separately. A Schrieffer-Wolff transformation of H allows us to obtain an effective Hamiltonian for the nuclear spins [7, 8],

$$H_n^{eff} = \frac{1}{2}\sum_{ij\alpha}\frac{J_{ij}^\alpha}{N_\perp^2}\hat{I}_i^\alpha\hat{I}_j^\alpha = \frac{1}{L}\sum_{q\alpha}\frac{J_q^\alpha}{N_\perp^2}\hat{I}_{-q}^\alpha\hat{I}_q^\alpha, \qquad (1)$$

where $\alpha = x, y, z$, and $J_{ij}^\alpha = A_0^2\chi_{ij}^{\alpha\alpha}a/2$ is the effective RKKY [25] interaction between nuclear spins. a is the lattice spacing and provides the short distance cutoff of the continuum theory. The sum over $q = n\pi/L$ for integer n runs over the first Brillouin zone. $\chi_{ij}^{\alpha\alpha} = -ia^{-1}\int_0^\infty dt\ \langle[\hat{S}_i^\alpha(t), \hat{S}_j^\alpha(0)]\rangle e^{-\eta t}$ (for an infinitesimal $\eta > 0$) is the static electron spin susceptibility. We also have defined $\hat{I}_q^\alpha = \sum_i e^{ir_iq}\hat{I}_i^\alpha$ and $J_q^\alpha = \int dr\ e^{-irq}J^\alpha(r)$.

The effective electron Hamiltonian, on the other hand, includes the effect of the feedback of the nuclear field on the electrons. Since the spins $\tilde{I} = IN_\perp$ are large, we can choose $H_{el}^{eff} = H_{el} + H_{Ov}$, with $H_{Ov} = \sum_i \mathbf{h}_i\cdot\hat{\mathbf{S}}_i$ and $\mathbf{h}_i = A\langle\hat{\mathbf{I}}_i\rangle$ the nuclear Overhauser field.

Interacting electrons as Luttinger liquid. — We use a bosonized Hamiltonian to describe the interacting electron system of the armchair SWNT, which is naturally in the Luttinger liquid state due to the linear electron dispersion [14, 15]. The unit cell of a graphite sheet contains two carbon atoms, which results into a two-band description of the bosonized system. Since mixing between the bands is essentially absent [14, 15] we shall, however, focus on a single band only in order to avoid a heavy notation. The bosonized single-band Hamiltonian reads [14, 15, 26]

$$H_{el} = \sum_{\nu=c,s}\int\frac{dr}{2\pi}\left[\frac{v_\nu}{K_\nu}(\nabla\phi_\nu(r))^2 + v_\nu K_\nu(\nabla\theta_\nu(r))^2\right], \qquad (2)$$

where $\phi_{c,s}$ are boson fields such that $-\nabla\phi_{c,s}\sqrt{2}/\pi$ express charge and spin density fluctuations, respectively. $\theta_{c,s}$ are such that $\nabla\theta_{c,s}/\pi$ are canonical conjugate to $\phi_{c,s}$. $v_{c,s} = v_F/K_{c,s}$ are charge and spin wave velocities, and $K_{c,s}$ are the dimensionless Luttinger liquid parameters. For SWNTs [14, 15], $K_c \approx 0.2$. If the electron spin SU(2) symmetry is maintained, $K_s = 1$, otherwise $K_s \neq 1$.

Without feedback from nuclear magnetic field. — Let us first assume that there is no feedback from the Overhauser field on the electrons and set $\mathbf{h}_i \equiv 0$. The electron system forms a Luttinger liquid, for which the zero temperature spin susceptibility has a singularity at momentum $q = \pm 2k_F$ induced by backscattering processes [26, 27]. At $T > 0$ this singularity turns into a steep but finite minimum: The backscattering part of the spin operator $\hat{S}_i^x$ is expressed in the bosonization language by the operators [26] $\hat{O}_{SDW}^x(r_i) \propto e^{-2ik_Fr_i}e^{i\sqrt{2}\phi_c}\cos(\sqrt{2}\theta_s)$, such that $\hat{S}^x = [\hat{O}_{SDW}^x + \hat{O}_{SDW}^{x\dagger}]/2$ plus forward scattering terms. Similar expressions [26] hold for $\hat{S}^y$ and $\hat{S}^z$. We further assume that $J_q^\alpha \equiv J_q$ is isotropic and in particular $K_s = 1$. The correlators between those operators can be evaluated in the standard way and we obtain (for $q > 0$)

$$J_q(g, v_F) \approx -C(g, v_F)(k_BT)^{2g-2}\left|\Gamma(\kappa)/\Gamma(\kappa+1-g)\right|^2, \qquad (3)$$

where $g = (K_c + K_s^{-1})/2$, $\kappa = g/2 - i\lambda_T(q-2k_F)/4\pi$, depending on the thermal length $\lambda_T = v_F/k_BT$ with k_B the Boltzmann constant. Γ is Euler's Gamma function and $C(g, v_F) = A_0^2 a\sin(\pi g)\Gamma^2(1-g)(2\pi a/v_F)^{2g-2}/8\pi^2 v_F$. We have made the inessential assumption $v_c = v_s = v_F$. Note that J_q is independent of k_F for a linear dispersion. A density dependence of J_q requires a curvature of the electron dispersion, which partially restores Fermi liquid properties [28], a scenario which we disregard for metallic SWNTs. A sketch of J_q is shown in Fig. 2.

At temperatures $T < T_0^*$ [defined in Eq. (6) below], $|J_{2k_F}(T)| > k_BT$ and the nuclear spins can – classically – minimize the RKKY energy by aligning in a spiral order $\mathbf{I}_i' = IN_\perp[\cos(2k_Fr_i)\mathbf{e}_x + \sin(2k_Fr_i)\mathbf{e}_y]$, where $\mathbf{e}_{x,y}$ are orthonormal vectors defining the spin (x, y) plane. We shall henceforth *assume* that this order is established, and show that this assumption is self-consistent. Fluctuations reduce this maximal polarization, and in gen-

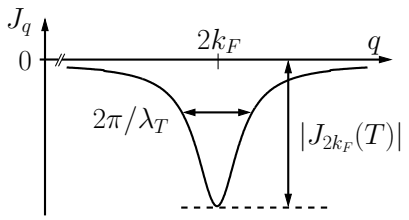

FIG. 2: Sketch of the RKKY interaction J_q given by Eq. (3).

eral $|\langle\hat{\mathbf{I}}_i\rangle| < IN_\perp$. The lowest lying excitations (to order $1/IN_\perp$) in the nuclear spin system are magnons. Since J_{ij} is long-ranged the energy cost of local defects, like kinks, scales with the system size and is very high.

For a helimagnet, there exists a gapless magnon band with the dispersion [8] $\omega_q = 2(IN_\perp)(J_{2k_F+q}/N_\perp^2 - J_{2k_F}/N_\perp^2)$. Let $m_i = \langle\hat{\mathbf{I}}_i\rangle \cdot \mathbf{I}_i'/(IN_\perp)^2$ measure the component of the average magnetization along $\mathbf{I}_i'$, normalized to $0 \le m_i \le 1$. Its Fourier component m_{2k_F} acts as an order parameter for the spiral phase. Magnons decrease this order parameter and we have [8]

$$m_{2k_F}(T) = 1 - \frac{a}{(IN_\perp)L}\sum_{q\neq 0}\frac{1}{\mathrm{e}^{\omega_q/k_BT}-1}, \qquad (4)$$

where the sum represents the magnon occupation number. In the continuum limit $L \to \infty$ the integrand is divergent as $1/q^2$ for $q \to 0$ (the $q = 0$ mode is absent because the system is not a ring), showing the absence of true long range order in the 1D system. Despite its appearance the divergence is not a consequence of the Mermin-Wagner theorem [29, 30], which forbids long range order in low-dimensional systems for sufficiently short ranged interactions. Since J_{ij} is long ranged this theorem does not apply.

The present situation, however, is very different in that the system has a finite length $L \sim 2$ μm imposed either through the natural length of the nanotube or through an external confining potential. At temperatures $T < T_0^*$ we find that $L \ll \lambda_T$, and so the cost of exciting the first magnon is already very high $\omega_{q=\pi/L} \approx 2I|J_{2k_F}(T)|/N_\perp$. We can define a temperature T_{M0} providing the scale of the excitation of the first magnons by imposing $\omega_q/k_BT \approx 2I|J_{2k_F}(T)|/N_\perp k_BT = 1$. For $T > T_{M0}$ we can then simplify Eq. (4) to

$$m_{2k_F}(T) \approx 1 - \frac{1/IN_\perp}{\mathrm{e}^{(\frac{T_{M0}}{T})^{3-2g}}-1} \approx 1 - \left(\frac{T}{T_0^*}\right)^{3-2g}, \quad (5)$$

where we have defined

$$k_BT_0^* = \left[2I^2C(g,v_F)\Gamma^2(g/2)\Gamma^{-2}(1-g/2)\right]^{\frac{1}{3-2g}}. \quad (6)$$

For the SWNT this temperature satisfies the self-consistency condition $k_BT_{M0} < k_BT_0^* \ll v_F/L$. We use T_0^* as an estimate for the critical temperature. For a typical SWNT T_0^* is very low. With the values given with Fig. 3 we obtain $T_0^* \sim 10$ μK, too low for experimental detection. Yet this analysis completely neglects the feedback of the magnetic field on the electron gas. This leads to a strong renormalization of T_0^*.

Feedback of nuclear magnetic field on electrons. — The ordering of the nuclear spins leads to a spatially oscillating Overhauser field $\mathbf{h}_i = A\langle\mathbf{I}_i\rangle$ that acts back on the electrons. We choose the electron spin axis such that $\hat{\mathbf{S}}\cdot\mathbf{e}_x = \hat{S}^x$ and $\hat{\mathbf{S}}\cdot\mathbf{e}_y = \hat{S}^y$. The spatial oscillations of $\mathbf{h}_i \propto \mathrm{e}^{\pm 2ik_Fr_i}$ in H_{Ov} perfectly cancel some of the spatial oscillations of the $\hat{O}_{SDW}^{x,y}$ operators of the $\hat{S}_i^{x,y}$. Neglecting the remaining (irrelevant) oscillating terms we obtain $H_{Ov} \approx \sum_i A_0 I m_{2k_F}\cos(\sqrt{2K}\phi_+(r_i))$, where we have introduced $\phi_+ = (\phi_c+\theta_s)/\sqrt{K}$ with the normalization $K = K_c + 1/K_s$. The Hamiltonian becomes of the sine-Gordon type and H_{Ov} is relevant in the sense of the renormalization group (RG): The ϕ_+ field is pinned at a minimum of the cosine term of H_{Ov}. The result is a density wave that combines charge and spin degrees of freedom. Fluctuations about the minimum are massive, with a mass associated to an energy scale Δ. At commensurate electron filling Umklapp processes would become relevant too, and lead to fully gapped charge and spin sectors. For SWNTs, however, this would require high electron densities leading to $E_F \approx 1.4$ eV. This case is not considered here.

Within a perturbative RG approach we find that

$$\Delta \sim (A_0 I m_{2k_F}/E_F)^{1/(2-g)} v_F/a. \qquad (7)$$

This mass gap Δ is the first important consequence of the feedback. The second important consequence is the spontaneous generation of anisotropy because the spin (x,y) plane is singled out by the Overhauser field. This is seen, for instance, in the spin susceptibilities $\chi^{\alpha\alpha}$. Those can be calculated in the same way as before (see Appendix A) if we notice that the massive ϕ_+ field does not contribute to the long-wavelength asymptotics. The finite temperature expressions for the $\chi^{\alpha\alpha}$ are otherwise identical to the case without feedback, and the RKKY couplings J_q^α can be obtained from Eq. (3) upon the following modifications: For χ^{xx} and χ^{yy} the exponent g is replaced by $g' = 2K_c/K_sK$ and the amplitude is reduced by a factor 2 because a term depending on ϕ_+ only drops out. For χ^{zz} the exponent becomes $g'' = (K_c/K_s + K_cK_s)/2K$ while the amplitude remains unchanged. v_F is replaced by $v_- = (v_c/K_c + v_sK_s)/K$. This leads to

$$J_q^{x,y} = J_q(g', v_-)/2, \; J_q^z = J_q(g'', v_-). \qquad (8)$$

For $K_c = 0.2$ and $K_s = 1$ we have to compare $g = 0.6$ with the strongly renormalized $g' = 0.33$ and $g'' = 0.17$.

Let us finally note that correlators between ϕ_+, θ_+ can only be neglected as long as $k_BT < \Delta$, i.e. $\lambda_T^{-1} < \xi^{-1}$ with $\xi = v_F/\Delta$ the correlation length. In Eq. (9) below we define a critical temperature T^* similarly to T_0^* before. For $T \ll T^*$, $m_{2k_F} \approx 1$ (see Fig. 3), and we find that $\Delta \gg k_BT$. At $T \to T^*$, however, m_{2k_F} vanishes and so does Δ. The order in electron and nuclear systems, therefore, vanishes simultaneously.

Consequences for magnetization and transport. — The helical order still minimizes the energy and there remains a gapless magnon band [8], $\omega_q = 2I(J'_{2k_F+q} - J'_{2k_F})/N_\perp$, where $J'_q = J^x_q = J^y_q$. The previous discussion of the magnetization remains otherwise unchanged. Replacing J_q by J'_q in Eq. (6) leads to the renormalized critical temperature T^*,

$$k_B T^* = \left[I^2 C(g', v_-)\Gamma^2(g'/2)\Gamma^{-2}(1-g'/2)\right]^{\frac{1}{3-2g'}}. \quad (9)$$

The notable difference is the modified exponent. For the parameters displayed with Fig. 3, we obtain the change from $1/(3-2g) = 0.625$ to $1/(3-2g') \approx 0.43$. Quite remarkably this considerably boosts the value of the characteristic temperature from $T_0^* \sim 10$ μK to $T^* \sim 1$ mK. Note that $T^* \ll v_F/Lk_B$ is still satisfied. Fig. 3 (**a**, solid line) shows the result of the feedback. In Fig. 3 (**b**) we also show the dependence of T^* on A_0, $T^* \propto A_0^{2/(3-2g')} = A_0^{0.86}$.

The order furthermore modifies the transport properties of the system. With the opening of the mass gap in the ϕ_+ channel, half of the conducting modes are blocked and the conductance decreases by the universal factor of 2. As an illustration we consider a SWNT connected to metallic leads. The conductance is given by [31, 32, 33] $G = 4e^2/h$, where e is the electron charge, h the Planck constant, and where 4 is the number of conducting channels (2 spin projections and 2 bands). The pinning of the ϕ_+ field (in each band) blocks 2 conductance channels and so reduces the conductance precisely by the factor 2 (see Appendix B for details). Such a reduction is a direct consequence of the nuclear spin ordering and the Luttinger liquid physics of the electrons, and should be detectable experimentally in standard transport setups.

As a conclusion, we emphasize that the physics described here is quite general and is also relevant for other 1D systems of the Kondo-lattice type.

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APPENDIX A: SUSCEPTIBILITIES IN THE PARTIALLY GAPPED SYSTEM

The static susceptibilities $\chi^\alpha(q) = \chi^\alpha(q, \omega = 0)$ are the Fourier transforms of the following retarded electron spin response functions (we set $\hbar = 1$ throughout this section)

$$\chi^\alpha(r,t) = -i\Theta(t)\langle[S^\alpha(r,t)\,,\, S^\alpha(0,0)]\rangle, \quad (A1)$$

where $S^\alpha(r,t)$ are the electron spin operators at position r and time t, and $\Theta(t)$ is the step function.

In the bosonization language, the spin operators are expressed through the sum of forward scattering and backscattering operators as [26] $S^\alpha = (S^\alpha)_{forw} + [O^\alpha_{SDW} + (O^\alpha_{SDW})^\dagger]/2$. The forward scattering operators lead to a contribution to χ^α that is regular in the momentum q, while the backscattering parts are singular or strongly peaked at momentum $q = 2k_F$. We shall therefore neglect the forward scattering contribution and focus only on the backscattering operators (not writing Klein factors) [26]

$$O^x_{SDW} = \frac{\mathrm{e}^{-2ik_F r}}{2\pi a}\mathrm{e}^{i\sqrt{2}\phi_c}\left[\mathrm{e}^{i\sqrt{2}\theta_s} + \mathrm{e}^{-i\sqrt{2}\theta_s}\right], \quad (A2)$$

$$O^y_{SDW} = i\frac{\mathrm{e}^{-2ik_F r}}{2\pi a}\mathrm{e}^{i\sqrt{2}\phi_c}\left[\mathrm{e}^{i\sqrt{2}\theta_s} - \mathrm{e}^{-i\sqrt{2}\theta_s}\right], \quad (A3)$$

$$O^z_{SDW} = i\frac{\mathrm{e}^{-2ik_F r}}{2\pi a}\mathrm{e}^{i\sqrt{2}\phi_c}\left[\mathrm{e}^{i\sqrt{2}\phi_s} - \mathrm{e}^{-i\sqrt{2}\phi_s}\right]. \quad (A4)$$

The average in Eq. (A1) is evaluated with respect to the following Hamiltonian, depending on the gapped ϕ_+ and ungapped ϕ_- fields,

$$H = \int \frac{dr}{2\pi}\Big\{v_-\left[(\nabla\phi_-(r))^2 + (\nabla\theta_-(r))^2\right] + v_+\left[(\nabla\phi_+(r))^2 + (\nabla\theta_+(r))^2 + \xi^{-2}\,\phi_+^2(r)\right]\Big\}, \quad (A5)$$

where $\xi = v_+/\Delta$ is the correlation length associated to the gap Δ, $v_+ = (v_c K_c + v_s/K_s)/K$ and $v_- = (v_c/K_s + v_s K_c)/K$ for $K = K_c + 1/K_s$.

The boson fields $\phi_\pm, \theta_\pm$ are related to the usual charge and spin boson fields through the transformation

$$\phi_c = \frac{\sqrt{K_c}}{\sqrt{K}}\left[\sqrt{K_c}\phi_+ - \frac{1}{\sqrt{K_s}}\phi_-\right], \quad (A6)$$

$$\phi_s = \frac{\sqrt{K_s}}{\sqrt{K}}\left[\frac{1}{\sqrt{K_s}}\theta_+ + \sqrt{K_c}\theta_-\right], \quad (A7)$$

$$\theta_c = \frac{1}{\sqrt{K_c K}}\left[\sqrt{K_c}\theta_+ - \frac{1}{\sqrt{K_s}}\theta_-\right], \quad (A8)$$

$$\theta_s = \frac{1}{\sqrt{K_s K}}\left[\frac{1}{\sqrt{K_s}}\phi_+ + \sqrt{K_c}\phi_-\right]. \quad (A9)$$

Let us set $\bar{r} = (r,t)$, $\bar{\chi}^\alpha(\bar{r}) = -i2(a\pi)^2\chi^\alpha(r,t)$, and assume that $t > 0$. We can then write, for instance for χ^x,

$$\bar{\chi}^x(\bar{r}) = \cos(2k_F r)\Big[\left\langle\left[\mathrm{e}^{i\sqrt{2K}\phi_+(\bar{r})}\,,\,\mathrm{e}^{-i\sqrt{2K}\phi_+(0)}\right]\right\rangle + \left\langle\left[\mathrm{e}^{i\sqrt{2K'}\phi_-(\bar{r})-i\sqrt{2K''}\phi_+(\bar{r})}\,,\,\mathrm{e}^{-i\sqrt{2K'}\phi_-(0)+i\sqrt{2K''}\phi_+(0)}\right]\right\rangle\Big], \quad (A10)$$

with $K' = 4K_c/K_s K$ and $K'' = (K_c - K_s^{-1})^2/K$, and where we have used the invariance of the Hamiltonian under a simultaneous sign change of all the boson fields. In the Gaussian theory (A5) the correlators in (A10) are fully expressed through boson correlators of the form $\langle\phi_\pm(\bar{r})\phi_\pm(0)\rangle$ which, in (q,ω) space, are given by [26]

$$\langle\phi^*_-(q,\omega)\phi_-(q,\omega)\rangle = \frac{\pi v_-}{(\omega \pm i\eta)^2 - v_-^2 q^2}, \quad (A11)$$

for the massless fields and

$$\langle\phi^*_+(q,\omega)\phi_+(q,\omega)\rangle = \frac{\pi v_+}{(\omega \pm i\eta)^2 - v_+^2 q^2 - \Delta^2}, \quad (A12)$$

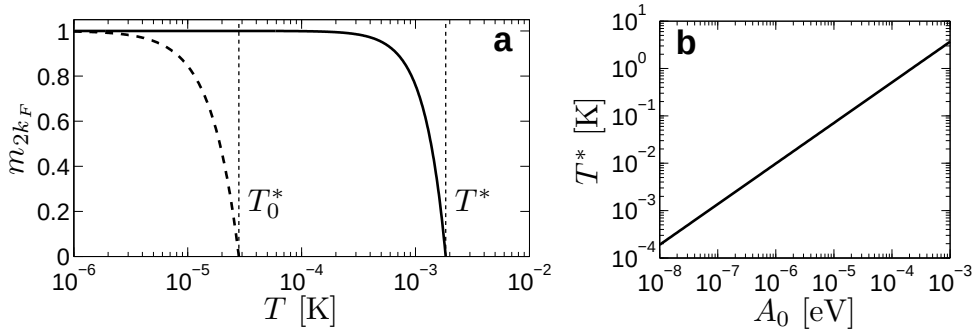

FIG. 3: **a**: Magnetization $m_{2k_F}(T)$ [Eq. (5)]. Dashed line: without feedback. Solid line: with feedback. Parameters for the curves are [14, 15, 23, 24] $E_F = 0.1$ eV, $A_0 = 10^{-7}$ eV, $v_F = 8 \times 10^5$ m/s, $a = 2.46$ Å, $K_s = 1$, $K_c = 0.2$ (leading to $g = 0.6$, $g' = 0.33$), and $L = 2$ μm. The vertical lines mark the temperatures written next to them. **b**: Characteristic temperature T^* [Eq. (9)] as a function of the hyperfine constant A_0. The curve follows a power law $T^* \propto A_0^{2/(3-2g')} = A_0^{0.86}$, and is plotted up to the self-consistency limit $T^* \approx v_F/Lk_B = 3$ K.

for the massive fields. The sign the infinitesimal shift $\pm i\eta$ is dictated by the time order of the operators. Eq. (A11) is singular at $\omega \to 0, q \to 0$, and the proper treatment of this singular behavior leads to the power law behavior of the susceptibilities (at the shifted $q \to \pm 2k_F$) characteristic for a Luttinger liquid theory.

On the other hand, Eq. (A12) is regular at $\omega \to 0, q \to 0$, and so does not contribute to the power law divergence at $q \to \pm 2k_F$. In fact, let us expand Eq. (A10) in powers of ϕ_+. The lowest nonzero term in the ϕ_+ is

$$\cos(2k_F r)\Big[2K\big\langle\big[\phi_+(\bar r)\,,\,\phi_+(0)\big]\big\rangle + 2K''\big\langle\big[\mathrm{e}^{i\sqrt{2K'}\phi_-(\bar r)}\phi_+(\bar r)\,,\,\mathrm{e}^{-i\sqrt{2K'}\phi_-(0)}\phi_+(0)\big]\big\rangle\Big]. \tag{A13}$$

The Fourier transform of the first term, depending on ϕ_+ only, at $\omega \to 0, q \to \pm 2k_F$ tends to a constant $\propto 1/\Delta^2$, and so contributes only insignificantly to the static susceptibility $\chi^x(q, \omega = 0)$ at $q \approx 2k_F$. The second term involves a sum of products of the type

$$\cos(2k_F r)\mathrm{e}^{K'\langle\phi_-(\bar r)\phi_-(0)\rangle}\langle\phi_+(\bar r)\phi_+(0)\rangle. \tag{A14}$$

Since ϕ_- is massless, the exponential factor evaluates to a power law. With Eq. (A12) we can then write the Fourier transform of Eq. (A14) as a convolution of the form

$$\int dq' d\omega' \left|\frac{1}{\omega'^2 - v_-^2 q'^2}\right|^{1-K'} \frac{1}{(\omega-\omega')^2 - v_+^2(q_\pm - q')^2 - \Delta^2}, \tag{A15}$$

where $q_\pm = q \pm 2k_F$. For $\omega \to 0$ and $q_\pm \to 0$ we see that the ω' integral is dominated by the poles at $\pm\sqrt{q'^2 + \Delta^2}$ and the weak singularities at $\pm q'$. The contribution at the poles is more singular, and if we focus on the pole at $\omega' = \sqrt{v_+^2 q'^2 + \Delta^2}$, we obtain

$$\sim \frac{1}{\Delta^{2(1-K')}} \int dq' \frac{1}{\sqrt{v_+^2 q'^2 + \Delta^2}}. \tag{A16}$$

The remaining integral leads to an arcsinh, which has an ultraviolet divergence that has to be cut off at $1/a$. More importantly, however, the result has no infrared divergence, meaning that this expression remains regular at $q_\pm \to 0$.

The latter results allow us to conclude that the Fourier transform of Eq. (A13) is regular at $q \to \pm 2k_F$. Since the theory (A5) is Gaussian, higher order correlators are products of the latter results and so remain regular. We have therefore shown that the singular behavior of the susceptibility is fully controlled by the ϕ_+ independent term in the expansion of the $\mathrm{e}^{\pm i\phi_+}$, allowing us to use the approximation

$$\bar\chi^x(\bar r) \approx \cos(2k_F r)\big\langle\big[\mathrm{e}^{i\sqrt{2K'}\phi_-(\bar r)}\,,\,\mathrm{e}^{-i\sqrt{2K'}\phi_-(0)}\big]\big\rangle, \tag{A17}$$

which is of precisely the same form as the susceptibility of a regular Luttinger liquid. The difference in the present case is the modified exponent K' and the fact that the first term in Eq. (A10), depending on ϕ_+ only, drops out. The amplitude of the resulting susceptibility $\chi^x(q)$ is reduced by a factor of 2. With $g' = K'/2$ this leads to the zero temperature susceptibility [26]

$$\chi^x(q) = -\frac{1}{2}\frac{\sin(\pi g')}{4v_-\pi^2}\Gamma^2(1-g')\sum_\pm \left|\frac{2}{a(q \pm 2k_F)}\right|^{2-2g'}. \tag{A18}$$

The susceptibility $\chi^y(q)$ involves the same combinations of the ϕ_+ and ϕ_- fields and is identical to $\chi^x(q)$. The susceptibility $\chi^z(q)$ is expressed through the combinations $\phi_c \pm \phi_s = \frac{1}{\sqrt{K}}\left(K_c\phi_+ - \sqrt{\frac{K_c}{K_s}}\phi_- \mp \theta_+ \mp \sqrt{K_c K_s}\theta_-\right)$. In contrast to χ^x and χ^y, the massless fields do not cancel out for one of the $\pm$ signs, and so the previous reduction of the amplitude by the factor 2 does not occur. Again we can neglect the contributions from the massive ϕ_+ and θ_+ fields. The resulting exponent of the power law in $\chi^z(q)$ then depends only on the combination of the prefactors of the ϕ_- and θ_- fields, and is given by

$g'' = (K_c/K_s + K_c K_s)/2K$ so that

$$\chi^z(q) = -\frac{\sin(\pi g'')}{4v_-\pi^2}\Gamma^2(1-g'')\sum_{\pm}\left|\frac{2}{a(q\pm 2k_F)}\right|^{2-2g''}. \tag{A19}$$

The extension to temperatures $T > 0$ is straightforward [26] and the result is given by Eqs. (3) and (8). The main effect is that the singularity at $q = \pm 2k_F$ turns into a finite minimum.

As temperature rises, the depth of this minimum decreases. The neglected ϕ_+ correlators become important when the temperature becomes comparable to Δ. For temperatures below T^*, the helical magnetization m_{2k_F} is close to 1 and so $\Delta \approx (AI/E_F)^{1/(2-g)}v_F/a$. For the values (see Fig. 3) $A/E_F = 10^{-6}, v_F = 8\times 10^5$ m/s, $a = 2.46$ Å and $g = K/2 = 0.6$, we obtain $\Delta \approx 6.8\times 10^{-5}$ eV, i.e. a corresponding temperature of 0.7 K. This is much higher than the critical temperature T^* and confirms the validity of the approximations above.

APPENDIX B: REDUCTION OF THE ELECTRICAL CONDUCTANCE

We illustrate here the reduction of the conductance of the nanotube by a universal factor 2 with the calculation of the electrical DC conductance G of a Luttinger liquid that is connected to metallic leads. It was shown in [31, 32, 33] that in such a system the conductance is given by $G = \frac{e^2}{h}n$, where n is the number of conducting channels in the Luttinger liquid. The Luttinger liquid properties appear in this formula only through the number n of channels, while the prefactor e^2/h is entirely determined by the properties of the leads.

A metallic single wall carbon nanotube has $n = 4$, which is composed of a factor 2 arising from the two spin directions, and a factor 2 because the unit cell has two carbon atoms leading to two electron bands. With the feedback from the ordered nuclear magnetic field the ϕ_+ field is pinned and we expect that it no longer contributes to the electrical conduction. Since there is a ϕ_+ in each band, the number of conducting channels then reduces to $n = 2$.

In the following we shall prove this intuitive result: As in [31, 32, 33] we model the leads by one-dimensional Fermi liquids with $K_c = K_s = 1$. The validity of the quadratic Hamiltonian (A5) can then be extended into the leads by introducing a spatial dependence on $v_\pm$ and ξ (or Δ) such that $v_\pm(r) = v_F$ and $\xi(r) = \Delta(r) = 0$ when r lies in the leads.

In the bosonization formulation the DC conductance can be calculated through the $\omega \to 0$ limit of the nonlocal conductivity [26]

$$\sigma(r,r';\omega) = -i\frac{4e^2}{h}(\omega + i\eta)g^r(r,r';\omega), \tag{B1}$$

where $g^r(r,r';\omega)$ is the Fourier transform of the retarded boson Green's function

$$\begin{aligned} g^r(r,r;t) &= -i\Theta(t)\langle[\phi_c(r,t)\,,\,\phi_c(r',0)]\rangle \\ &= L_+(r,r')g^r_+(r,r';t) + L_-(r,r')g^r_-(r,r';t), \end{aligned} \tag{B2}$$

with

$$g^r_\pm(r,r;t) = -i\Theta(t)\langle[\phi_\pm(r,t)\,,\,\phi_\pm(r',0)]\rangle \tag{B3}$$

and

$$L_+(r,r') = \frac{K_c(r)K_c(r')}{\sqrt{K(r)K(r')}}, \tag{B4}$$

$$L_-(r,r') = \sqrt{\frac{K_c(r)K_c(r')}{K_s(r)K_s(r')K(r)K(r')}}. \tag{B5}$$

As found in [31, 32, 33] the conductance depends only on the properties of the leads, where $L_\pm(r,r') = 1/2$. If the system were entirely gapless, both ϕ_- and ϕ_+ channels would therefore contribute equally to the total conductance, with $2e^2/h$ from each channel (the latter factor 2 being due to the two bands).

The gap Δ affects only the ϕ_+ fields, and so the contribution $2e^2/h$ from the two ϕ_- channels remains unchanged. The contribution from the gapped ϕ_+ fields, however, drops to zero: If we follow the method of Ref. [31] the inclusion of the finite gap $\Delta(r)$ in the calculation of g^r_+ is straightforward. The result is

$$g^r_+(r,r';\omega) = \frac{1}{2\sqrt{\Delta^2-\omega^2}}\mathrm{e}^{-|r-r'|\sqrt{\Delta^2-\omega^2}/v_+}, \tag{B6}$$

not writing further regular terms expressing reflections of propagating waves on the ends of the nanotube, and for r and r' both lying in the nanotube. In the DC limit $\omega \to 0$ we then see that the corresponding conductance is zero,

$$(\omega + i\eta)g^r_+(r,r';\omega) \to 0, \tag{B7}$$

while ungapped fields have here the nonzero limit $i/2$. Hence the DC conductance is entirely determined by the two ϕ_- channels from the two bands of the nanotube and is given by

$$G = 2\frac{e^2}{h}. \tag{B8}$$

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